Simulation of a piezoelectric loudspeaker for hearing aids and experimental validation

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Introduction: The use of piezoelectric materials in hearing aid loudspeakers, also called receivers, presents technical and economic advantages such as reducing the number of parts of the system and its manufacturing cost. However, the performance of such systems is still not competitive when compared to traditional electrodynamic loudspeakers. In order to achieve an appropriate performance, one option is to apply optimization techniques to these systems. In optimization procedures, it is convenient to use efficient models to quickly evaluate the system performance, so that the evaluation can be repeated several times. Therefore, the aim of this work is to develop and validate an efficient multi-physical model of a hearing aid piezoelectric loudspeaker so that the model may be used in an optimization procedure.

Computational Methods: The performance of a hearing aid loudspeaker is usually evaluated through an experimental set-up where the loudspeaker is connected to a standard microphone coupler which simulate the ear canal impedance and provides an approximation of the incident sound pressure at the ear drum. An overview scheme of the model used to simulate this experimental set-up can be visualized in Figure 1, where the loudspeaker is represented by a multi-physical Finite Element (FE) model and the acoustic coupler as a Transfer Matrix Method (TMM) model [1] to reduce the computational cost of the analysis.
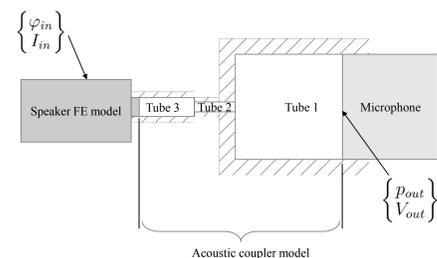

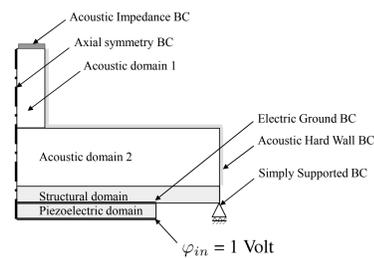**Figure 1**. Overview scheme of the speaker performance analysis model.

Figure 2. Multi-physical speaker FE model set-up.

Figure 2 presents the domains and boundary conditions (BC) applied to the loudspeaker FE model. As the piezoelectric loudspeaker has a cylindrical shape, to reduce the computational cost, a 2D axisymmentric FE model was used in COMSOL. The analysis was performed using the Acoustic-Piezoelectric Interaction physics module and the Frequency Domain study step. The TMM model is coupled to the FE model by means of the acoustic impedance BC.

The acoustic model of the small cavities of the loudspeaker accounts for thermal and viscous effects on the acoustic propagation. These effects may be modeled by the Thermoacoustics module of COMSOL, but this approach is computationally expensive. Therefore, these effects are included in a simplified form using the Low Reduced Frequency (LRF) [2,3] model by using Pressure Acoustic module of COMSOL.

The FE model physics are considering the following differential equations:

- **Piezoelectric Model**:
$$\rho\omega^2\boldsymbol{u} + \boldsymbol{\nabla}[\boldsymbol{\sigma}] + \boldsymbol{f_b} = 0$$
$$\boldsymbol{\nabla}\cdot\boldsymbol{D} - q_b = 0$$

Constitutive Relations:
$$[\boldsymbol{\sigma}] = [\boldsymbol{\sigma}](\boldsymbol{u},\varphi)$$
$$\boldsymbol{D} = \boldsymbol{D}(\boldsymbol{u},\varphi)$$

- **Structural Model**:
$$\rho\omega^2\boldsymbol{u} + \boldsymbol{\nabla}[\boldsymbol{\sigma}] + \boldsymbol{f_b} = 0$$

Constitutive Relation:
$$[\boldsymbol{\sigma}] = [\boldsymbol{\sigma}](\boldsymbol{u})$$

- **Acoustic LRF Model**:
$$B(s)\nabla^2 p - D'(s,Pr)k_0^2 p = 0$$

Analytical LRF functions:
$$B(s) = B(l,\omega,\rho_0,\mu)$$
$$D(s,Pr) = D(l,\omega,\rho_0,\mu,\kappa,C_p)$$

The acoustic coupler was considered as three coupled acoustic tubes, where each tube can be represented by the following transfer matrix with LRF functions to include the viscothermal effects.

$$\begin{Bmatrix} p_2 \\ V_2 \end{Bmatrix} = [M] \begin{Bmatrix} p_1 \\ V_1 \end{Bmatrix}$$

$$[M] = \begin{bmatrix} \cos(kL) & -\frac{Z}{iS}\sin(kL) \\ \frac{iS}{Z}\sin(kL) & \cos(kL) \end{bmatrix}$$

$$k = \frac{k_0}{i}\sqrt{\frac{D'(s,Pr)}{B(s)}}$$

$$Z = \frac{\rho_0 c_0}{i}\frac{1}{\sqrt{D'(s,Pr)B(s)}}$$

With the acoustic coupler transfer matrix and the microphone impedance it is possible to obtain the impedance of these systems to include as a boundary condition on the FE model. After solving the FE model it is possible to get the acoustic pressure at the microphone surface with the following relation and then obtain the sound pressure level (SPL).

$$p_{out} = \frac{p_{FE}}{M_{11} + M_{12}/Z_{mic}}$$

Results: The FE model of the loudspeaker was experimentally validated through tests with a prototype designed and built with dimensions larger than those of a hearing aid loudspeaker designs to allow its construction. Figure 3 shows the SPL results compared with experimental result measured in [4] and two commercial hearing aid loudspeakers (Knowles) measured with the same acoustic coupler.
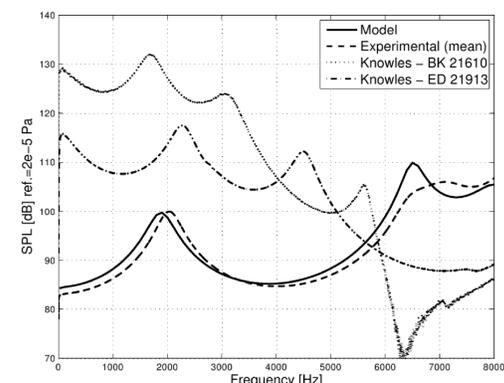

Figure 3. Comparison of numerical and experimental SPL of a piezoelectric prototype and two commercial hearing aid speakers (for an unit input spectra of electric potential).

Conclusions: This poster has presented a multi-physical model to analyze the performance of piezoelectric loudspeaker for hearing aid application. The model was simplified to increase its efficiency and to allow the application of optimization procedures. Despite the model simplifications, it was experimentally validated showing good agreement with the experimental results.

References:
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